ABSTract

The relation between time and space is difficult to describe precisely because we cannot measure time directly and can only make inferences based on how the space between objects changes from one measurement to the next. Here we describe a geometry of three imaginary dimensions of time, three real dimensions of space and a second power relation where space expands constantly and quadratically as a function of time, independently of the matter and energy content. In this model, the global geometry of spacetime is curved. This tiny but ubiquitous acceleration, when incorporated into the laws of motion, explains the least understood properties of our universe without a dependency on exotic physics: the accelerated expansion of the universe, the uniform temperature of the night sky, the velocity curves of spiral galaxies, and angular scale of the power spectrum of cosmic background radiation. Here we demonstrate that an assumption of globally curved spacetime matches the observed data better – and with fewer ad hoc theories – than an assumption of a flat universe.

*Keywords:* cosmology: large-scale structure of universe – cosmology: theory – galaxies: kinematics and dynamics

# Introduction

Theories based on an assumption of globally flat spacetime fail when applied to structures larger than our solar system. General Relativity does not predict the uniform temperature of the night sky (Rindler 1956). Stars and galaxies move too quickly in their orbits to be bound by Newton’s Laws of Gravitation (Rubin & Ford Jr 1970; Zwicky 1933). The Friedman-Robertson-Walker Metric didn’t predict the dimness of highly redshifted supernovae (Perlmutter et al. 1998; Riess et al. 1998). The Second Law of Motion in gravity bound systems predicts a relationship between the mass of a galaxy and the product of the radius and the second power of the orbital velocity. Instead, a fourth power relationship to the velocity – with no correlation to the radius – is observed. These failures bring us to a crisis where the ΛCDM model no longer even agrees with itself (Di Valentino et al. 2020). Rather than methodically examine the assumptions on which all these theories are based and look for a common misconception, the present landscape of science is littered “new physics” that instead try to fill the enormous gulf between the predictions and the observations with ideas that are increasingly untethered to reality.

Here we are not motivated by a desire to discover new physics, but by a desire to deconstruct the old physics. The common failure of all these theories is that they assume that the curvature of spacetime is so small that it can be ignored. Here we demonstrate that this faulty assumption is the root cause of the present crisis in Cosmology. Just as Foucault’s Pendulum was incontrovertible evidence that the Earth rotated about an axis, the Tully-Fischer Relation is incontrovertible evidence that the local geometry of spacetime is curved everywhere. This paper describes a new metric and a new set of properties based on this hypothesis and demonstrates that the predictions of this model match the observations better than the models employing the Lorentzian assumption.

# Chromatic Spacetime

Here we propose that there are three imaginary dimensions of time. Instead of the usual integer index, we are going to label them as red time, , green time, and blue time, . Like all imaginary numbers, we assume that none of these dimensions are directly measurable. Next, we are going to hypothesize that there is another set of dimensions that are the cross products of each pair of time dimensions. These are the dimensions of squared time (or space) and can be measured directly. There are three possible cross products. The product of red time and green time is yellow squared time, . The product of red time and blue time is magenta squared time, . The product of green time and blue time is cyan squared time, .

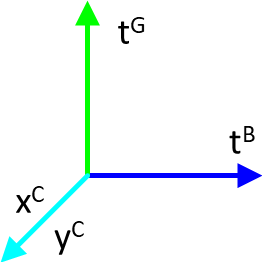
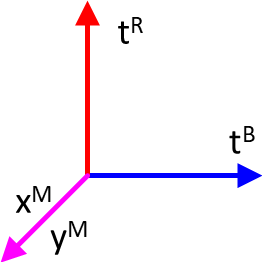
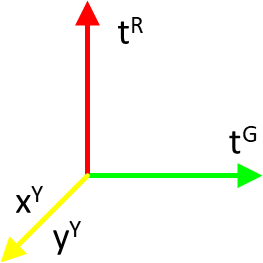


Figure 1 The three triplets of dimensions making up six-dimensional spacetime. The cross product of each pair of imaginary time dimensions produces a real dimension of space.

Figure 1 shows the relation between the dimensions schematically. These relations can be expressed mathematically as:

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| --- | --- | --- |
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|  |  |  |
|  |  |  |

The rules of this geometry are simple. If an observer were to move from the time coordinates to (in seconds), they would see a massless object in yellow squared time to move ( units. Note that this geometry is fundamentally asymmetric and space coordinates are not possible.

Like all imaginary numbers, time is only real and can only be measured directly when squared. There is no difference between squared time and space as they both refer to the same dimension. However, a method of converting squared time into practical units of space is required if we want to make predictions and test our hypothesis against the real world:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

Where , , and are the yellow, magenta, and cyan dimensions in some practical unit of space and is the conversion factor between that practical unit and squared time. For example, per . The subscript on the conversion factor indicates that this is the conversion for a single spacetime triplet only and is distinct from the aggregate conversion factor for three triplets that we will define shortly.

In the special case where and , the red and green time dimensions can be combined into yellow time and we can write:

|  |  |  |
| --- | --- | --- |
|  |  | () |

We can now state how yellow space changes with respect to yellow time in this special case:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |

Here we introduce the constant which describes the acceleration of the expansion of yellow space with respect to yellow time. Because of the second derivative of this relationship is not zero, the local geometry of this spacetime is curved everywhere. When each time coordinates in a spacetime triplet advance at the same rate, then all neutral, near massless objects in that manifold will irresistibly move away from each other at a constant acceleration of in their respective spatial dimensions.

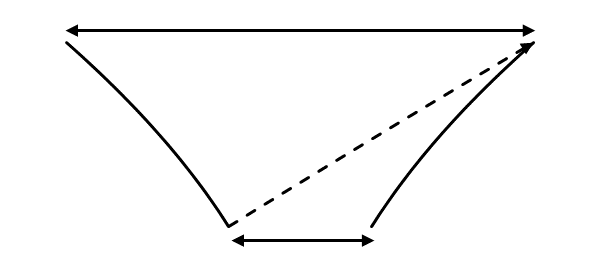
Another important property of this geometry is that constant velocity is unnatural. Everything accelerates with the expansion, even photons:

|  |  |  |
| --- | --- | --- |
|  |  | () |

Where is the speed of light at time , is the speed of light at , and represents the tangential velocity of the expansion. This is no different – and does not violate causality – any more than Hubble Expansion does. We have no problem believing that a galaxy, after some billions of years, can be travelling at twice the speed of light with respect to an observer. This formula simply says that the effects of expansion are not limited to only to distant objects, that the local geometry observes the same rules as global geometry.

# Metric Expansion

The primary observables that describe the evolution of the universe come from photons travelling from distant parts of the universe. Figure 2 describes the path of a photon as it travels through our hypothetical spacetime.



DC

DP

½DE

½DE

DLT

Figure 2. The path of a photon as it travels through quadratically expanding space. DP is the proper distance between the emitter and observer when the photon was emitted, DC is the comoving distance at the time of observation, DE is the amount of expansion between emission and observation, and DLT is the distance the photon has travelled.

The first step in calculating the light travel path is to determine the scale factor between a proper distance, , at some time, , when the photon is emitted and the comoving distance, , at some time in the future, , when the photon is observed. This scale factor is the same for all three sets of spacetime triplets, so we will dispense with the dimension index for now.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  | (5) |

Note that the scale factor is a quadratic function of time and time alone. The expansion of space during this trip, , is simply the difference between the comoving distance and the proper distance:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | () |

Another property of our hypothetical universe is that it is isotropic, which means that measurements should yield the same result no matter the direction. This is important as we assume that space expands equally in front of the photon as it does behind it during the trip, so that the light travel distance is simply the comoving distance less one half of the total expansion (see Figure 2).

|  |  |  |
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We now have enough information to describe the line elements for each of the spacetime triplets.

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |
|  |  | () |

These line elements combine to form the metric for six dimensional spacetime:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | () |

For the remainder of this paper, we are going to consider only the special case where all three dimensions of time have the same value and increase at the same rate for the observer:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

In addition, the constants for the individual spacetime triplets, and , can be combined into three-dimensional aggregates:

|  |  |  |
| --- | --- | --- |
|  |  |  |

The metric for six dimensions can now be consolidated into four dimensions as:

|  |  |  |
| --- | --- | --- |
|  |  | () |

None of our observations will employ cartesian coordinates, so it is useful to convert this metric to hyperspherical coordinates for positively curve spacetime:

|  |  |  |
| --- | --- | --- |
|  |  | (13) |

# Age and Acceleration

Because of the uniform mass of the progenitor star, type Ia Supernovae (SNe Ia) can be used to measure spacetime distances on cosmological scales. There are two observables from these stars that make this measurement possible: the change in photon’s wavelength, the redshift, tells us how space has expanded since the photon was emitted and the number of photons passing through a detector during a span of time from a known luminosity is a proxy for the spatial distance. We can use this information to test our hypothesis if we parameterize the hyperspherical metric from Eq. (13) to use redshift. The relation between the redshift and scale factor is:

|  |  |  |
| --- | --- | --- |
|  |  | () |

Light follows a null geodesic in spacetime, so we set , and the and terms are also set to zero since the path of a photon is a line-of-sight measurement.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  | (15) |

This is our formula for comoving distance as a function of redshift. However, luminosity is a power measurement – a count of photons per area per time – so we must account for the fact that these photons are traveling faster now than when emitted and therefore the frequency of detection will be lower by a factor of .

|  |  |  |
| --- | --- | --- |
|  |  | () |

Now that we have a formula that can predict the luminous distance, , to an object given the redshift, with enough data we can solve for the two unknowns - the age of the universe and the acceleration constant - using curve fitting. Using the combined data from (Conley et al. 2010), (Rodney et al. 2012), (Jones et al. 2013), (Rodney et al. 2016) and the parameters from (Rodney et al. 2016) to normalize the set of 476 SNe Ia data points, curve fitting tells us that the age of the universe, , is , and the acceleration constant, is. The Hubble constant,  , is . The reduced on the Quadratically Expanding Space (QES) model of 0.65 is a significantly better fit to the SNe Ia data than the ΛCDM model which has a reduced of 0.87 using the (Planck Collaboration et al. 2019) parameters of H0=67.4, and two free parameters, ΩΛ =0.685, Ωm = 0.315.

The difference between the models can be seen in Figure 3. The QES model predicts that SNe Ia are increasingly dimmer than the ΛCDM model predictions as redshift increases. Note that QES does not require exotic physics, has fewer free parameters, and still fits the SNe Ia data better than ΛCDM.

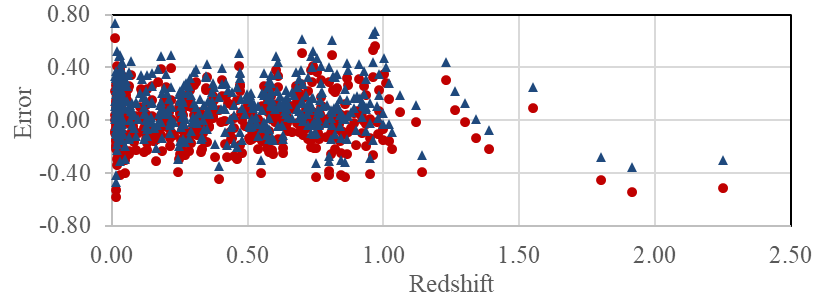
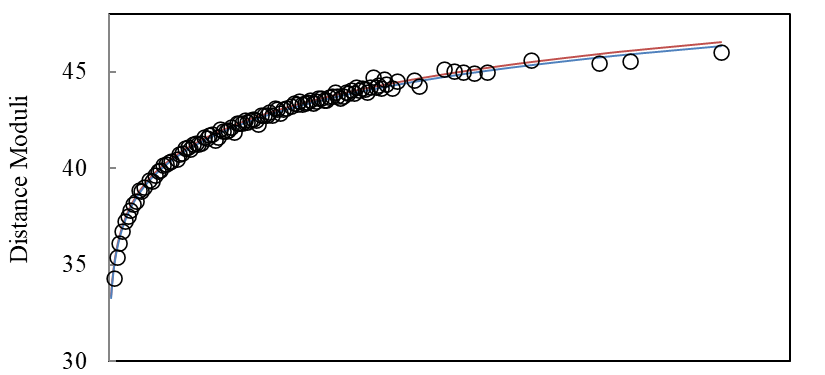


Figure 3. Above: Observed distance moduli (circles) are plotted with the QES predictions (red) and the ΛCDM predictions (blue). The formula used for the distance moduli is 5log(DL)+25 (DL in units of Mpc). Below: The errors for QES (red circle) and ΛCDM (blue triangle) in distance moduli.

# Spiral Galaxies

One of the profound implications of space that expands quadratically as a function of time is that there is a ubiquitous acceleration of . The rules of motion in this universe are radically different from the rules that Newton deduced three centuries ago. Objects at rest accelerate. Objects in motion accelerate. In fact, objects can only remain at rest when there is an unbalancing force keeping them there, such as gravity and electromagnetism. For a constant mass system at non-relativistic velocities, the sum-of-forces formula is:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | (17) |

Where is the velocity of the object, is the acceleration due to the imbalance of forces. is the scalar acceleration of expansion, and is a vector parallel to the line of force with a magnitude of .

The effects of quadratic expansion would be measurable in low acceleration environments where . Luckily, large spiral galaxies provide such a domain. We can predict the velocity of an object in a circular orbit knowing the enclosed mass and the distance from the center by applying the force of gravity and the acceleration of centripetal motion to Eq. (17) to get:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | (18) |
|  |  | (19) |

Where, for a given radius, *r, M(r)* is the enclosed mass, *v* is the tangential velocity and *G* is the gravitational constant. A mass model of a spiral galaxy is required to compare the prediction of Eq. (19) against the ΛCDM predictions. For QES, the model consists of a bulge and a disk. For ΛCDM, the model contains a bulge, a disk, and a halo of Dark Matter. The bulge mass in both models is assumed to have a de Vaucouleurs profile (Sofue et al. 2009):

|  |  |  |
| --- | --- | --- |
|  |  | () |

Where is the mass of the bulge, is 7.6695 and is the density at the scale radius, . Because an exact solution is computationally expensive and approximations are inaccurate, a deprojection table from (Young 1976) was used in the search for the best-fit parameters for the bulge mass.

The mass for an exponentially thin disk, , for both models is calculated from (Freeman 1970) as:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | () |

where is the central surface density, is the scale length of the disk.

The NFW profile (Navarro 1996) is a good model for the mass of the Dark Matter halo, , in the outer regions of galaxies. Integrating the density profile, the halo mass can be expressed as:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | () |

where is the characteristic density and is the scale radius of the halo.

Both models are fitted to the data from (Sofue et al. 2009) for the Milky Way Galaxy, (Chemin et al. 2009) for NGC 0224, (Sofue et al. 1999), (Sofue et al. 2003), (Sofue 2016), (Garrido et al. 2005), (Noordermeer et al. 2007), and (Martinsson et al. 2013). The best-fit ΛCDM model parameters are taken from (Sofue 2016). A sample of the resulting rotational curves can be found in Figure 4.

The results of comparing the best-fit parameters of the two models against the observed data are collected in Table 1. Out of the 43 galaxies selected for the (Sofue 2016) study, three galaxies (NGC 5533, UGC 02916, UGC 11852) were rejected because the velocities of gas in the outer disks were incompatible with regular motion. Except for the Milky Way and NGC 0224, the errors for the observed data were assumed to be unity (1 km s-1) and the modified *Χ2* method described in (Sofue 2016) was used (which is, essentially, a least-squares method). Of the 40 galaxies, the observed data in 29 were better matches to the QES predictions than the ΛCDM predictions. In aggregate, the QES model is a significantly better match with a average modified of 282.08 compared to ΛCDM with a average modified of 335.55. Note again that QES does not require exotic physics, has fewer free parameters, and still fits the rotation curve data better than ΛCDM.

|  |  |  |
| --- | --- | --- |
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Figure 4. A sampling of the tangential velocity predictions for QES (red) and ΛCDM (blue) and the observed data (gray). All horizontal axes are radius in kpc. Vertical axes are tangential velocity in km s-1.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | ΛCDM | | | | | | | QES | | | | |
| Name | ab | Mb | ad | Md | *h* | Mh | *Χ2* | ab | Mb | ad | Md | *Χ2* |
|  | (kpc) | (1010 M⊙) | (kpc) | (1010 M⊙) | (kpc) | (1010 M⊙) |  | (kpc) | (1010 M⊙) | (kpc) | (1010 M⊙) |  |
| Milky Way | 0.52 | 1.65 | 3.19 | 3.41 | 12.50 | 5.02 | 10.92 | 0.41 | 1.13 | 3.45 | 2.21 | 4.07 |
| M 31 | 1.30 | 3.36 | 4.30 | 7.70 | 30.50 | 27.90 | 27.85 | 0.00 | 0.00 | 3.36 | 4.60 | 24.39 |
| NGC 0253 | 0.93 | 1.60 | 1.90 | 1.90 | 7.90 | 3.50 | 326.48 | 0.63 | 0.84 | 2.34 | 1.76 | 163.95 |
| NGC 0266 | 2.89 | 2.12 | 6.60 | 25.10 | 87.10 | 239.30 | 1,545.58 | 25.40 | 22.31 | 12.42 | 22.49 | 897.72 |
| NGC 0342 | 0.64 | 0.52 | 1.60 | 1.60 | 12.00 | 7.90 | 91.78 | 0.00 | 0.00 | 3.36 | 2.87 | 15.41 |
| NGC 0598 | 7.66 | 0.22 | 6.10 | 0.50 | 8.50 | 2.40 | 29.69 | 0.00 | 0.00 | 1.61 | 0.15 | 49.74 |
| NGC 0660 | 0.57 | 0.94 | 0.60 | 0.20 | 9.20 | 3.80 | 25.34 | 0.30 | 0.44 | 0.96 | 0.42 | 231.53 |
| NGC 0891 | 0.71 | 1.84 | 3.10 | 3.10 | 6.40 | 3.10 | 313.24 | 0.64 | 1.27 | 2.65 | 2.05 | 334.39 |
| NGC 1365 | 0.90 | 2.51 | 3.30 | 6.60 | 14.40 | 10.40 | 439.67 | 0.75 | 1.56 | 3.48 | 4.24 | 228.46 |
| NGC 1642 | 3.06 | 2.78 | 3.00 | 6.00 | 88.90 | 83.10 | 260.34 | 0.00 | 0.00 | 2.15 | 2.28 | 260.16 |
| NGC 1808 | 0.66 | 1.38 | 3.00 | 3.20 | 2.00 | 0.40 | 227.54 | 0.75 | 1.23 | 1.86 | 0.91 | 171.48 |
| NGC 2403 | 0.14 | 0.02 | 0.20 | 0.00 | 7.60 | 2.90 | 42.32 | 0.58 | 0.07 | 1.85 | 0.29 | 90.21 |
| NGC 2543 | 0.99 | 0.15 | 3.80 | 4.00 | 20.60 | 14.10 | 337.91 | 4.38 | 0.47 | 6.40 | 3.96 | 181.91 |
| NGC 2599 | 1.08 | 10.03 | 3.50 | 12.50 | 50.20 | 43.80 | 1,314.98 | 0.35 | 1.32 | 1.21 | 5.07 | 1,731.39 |
| NGC 2649 | 1.56 | 0.16 | 3.10 | 3.10 | 19.70 | 11.70 | 124.38 | 0.00 | 0.00 | 3.74 | 2.04 | 10.23 |
| NGC 2654 | 1.07 | 1.45 | 2.80 | 5.70 | 52.90 | 38.20 | 346.30 | 0.44 | 0.36 | 2.32 | 2.13 | 263.34 |
| NGC 2903 | 2.50 | 5.24 | 3.80 | 8.00 | 7.60 | 5.00 | 670.73 | 4.71 | 8.70 | 4.04 | 4.09 | 363.78 |
| NGC 2985 | 0.56 | 0.39 | 1.10 | 1.10 | 9.70 | 4.20 | 25.14 | 0.54 | 0.37 | 1.29 | 0.61 | 894.78 |
| NGC 3079 | 0.69 | 2.63 | 3.80 | 4.40 | 16.20 | 9.50 | 477.67 | 0.74 | 2.25 | 4.12 | 2.86 | 247.47 |
| NGC 3198 | 6.21 | 0.50 | 3.10 | 1.60 | 13.40 | 6.20 | 165.51 | 0.00 | 0.00 | 3.12 | 1.01 | 375.89 |
| NGC 3521 | 0.72 | 1.52 | 1.60 | 3.50 | 22.60 | 14.00 | 406.18 | 0.06 | 0.17 | 1.50 | 2.01 | 83.32 |
| NGC 3628 | 0.84 | 1.52 | 3.60 | 3.60 | 8.50 | 4.80 | 170.05 | 1.31 | 1.97 | 4.78 | 3.15 | 34.77 |
| NGC 3900 | 1.66 | 2.44 | 6.30 | 11.90 | 7.70 | 2.80 | 252.55 | 1.54 | 1.64 | 4.06 | 3.15 | 58.30 |
| NGC 3982 | 0.55 | 0.09 | 1.20 | 1.20 | 23.60 | 13.80 | 102.88 | 1.20 | 0.06 | 1.54 | 0.73 | 9.86 |
| NGC 4258 | 0.47 | 1.19 | 0.80 | 0.90 | 19.40 | 16.10 | 209.35 | 1.75 | 4.14 | 5.63 | 0.80 | 116.09 |
| NGC 4303 | 0.33 | 0.15 | 2.10 | 1.10 | 11.90 | 4.50 | 318.52 | 0.27 | 0.11 | 1.96 | 0.54 | 518.35 |
| NGC 4321 | 1.32 | 2.76 | 7.60 | 16.80 | 7.10 | 4.20 | 347.82 | 1.72 | 2.92 | 6.44 | 6.89 | 97.08 |
| NGC 4527 | 0.41 | 0.94 | 2.60 | 1.70 | 12.20 | 7.40 | 502.75 | 0.45 | 0.95 | 4.09 | 2.20 | 426.68 |
| NGC 4565 | 3.05 | 6.40 | 4.00 | 4.60 | 19.20 | 16.30 | 551.64 | 1.69 | 1.59 | 3.04 | 3.87 | 478.89 |
| NGC 4569 | 1.39 | 0.66 | 12.00 | 39.70 | 9.80 | 3.50 | 892.04 | 2.90 | 1.24 | 30.00 | 54.55 | 448.85 |
| NGC 4736 | 0.82 | 1.09 | 0.90 | 0.80 | 7.30 | 2.30 | 212.82 | 1.88 | 2.57 | 1.30 | 0.24 | 149.87 |
| NGC 4945 | 0.36 | 0.69 | 0.30 | 0.30 | 9.00 | 5.60 | 92.88 | 0.75 | 1.53 | 3.84 | 1.08 | 113.01 |
| NGC 5033 | 1.04 | 3.76 | 5.70 | 11.40 | 38.90 | 41.20 | 421.40 | 0.65 | 1.90 | 5.07 | 6.05 | 90.16 |
| NGC 5055 | 2.96 | 4.15 | 1.80 | 1.70 | 7.70 | 4.20 | 385.20 | 2.22 | 1.38 | 1.89 | 1.54 | 389.59 |
| NGC 5236 | 0.19 | 0.47 | 3.00 | 1.80 | 8.10 | 4.40 | 249.21 | 0.17 | 0.40 | 2.66 | 1.33 | 780.03 |
| NGC 5457 | 2.76 | 2.90 | 2.40 | 2.20 | 8.10 | 4.60 | 426.31 | 2.10 | 1.26 | 2.81 | 2.39 | 305.37 |
| NGC 5907 | 1.60 | 2.76 | 6.90 | 13.80 | 6.70 | 3.40 | 402.47 | 1.72 | 2.08 | 4.99 | 5.09 | 112.49 |
| NGC 6946 | 0.36 | 0.92 | 3.80 | 4.20 | 9.40 | 5.60 | 199.42 | 0.36 | 0.80 | 4.27 | 3.26 | 118.68 |
| NGC 7013 | 0.82 | 0.64 | 0.80 | 0.80 | 11.80 | 5.70 | 229.69 | 1.87 | 0.98 | 1.07 | 0.54 | 193.43 |
| UGC 03993 | 4.35 | 16.59 | 4.80 | 7.50 | 135.10 | 214.90 | 245.45 | 1.50 | 3.55 | 3.33 | 4.86 | 218.01 |
| Average |  |  |  |  |  |  | 335.55 |  |  |  |  | 282.08 |

Table 1. Comparing the ΛCDM galaxy models and rotation curve predictions against the QES predictions.

# Fundamental Plane

There is a well-established empirical relation between tangential velocity (spiral) or central velocity dispersion (elliptical) of a galaxy and its luminosity (or mass). Astronomers have used this relation for decades to work out distance ladders, but there is no accepted theory to explain why this should be. Newtonian dynamics describes the mass of a galaxy as:

|  |  |  |
| --- | --- | --- |
|  |  | (23) |

Where is the mass inside radius and is the tangential velocity. This equation informs us that any combination of tangential velocity or radius is allowed, so according to the classical laws of dynamics, there simply is not enough information to draw any conclusion about the mass from the velocity parameter alone. What we observe, instead, is that mass (or luminosity) is related to the fourth power of the velocity, independent of the radius:

|  |  |  |
| --- | --- | --- |
|  |  | () |

Eq. (18) can be rearranged to calculate the mass of a system in quadratically expanding space as a function of the tangential velocity and radius:

|  |  |  |
| --- | --- | --- |
|  |  | (25) |

Given a velocity, the maximum mass will be found at radius, r:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  | (26) |

Substituting the equation (26) back into equation (25) yields the formula for the maximum mass given the velocity:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  | (27) |

Where the tangential velocity, , is in units of km s-1 and the maximum mass, , is in units of M⊙. Figure 5 shows that Classical Mechanics has no large-scale structure while QES exhibits large scale structure matching the fourth power of tangential velocity, independent of the radius.

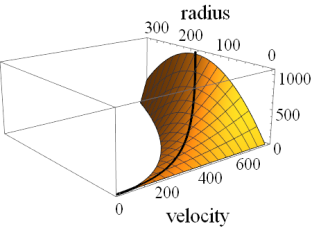
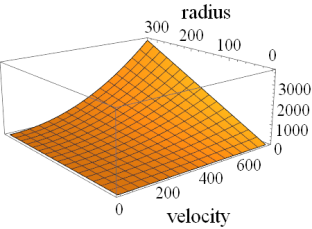


Figure 5. The Fundamental Plane. Radius is in km, tangential velocity in km s-1 and the vertical axis is in M⊙. Left: Classical Mechanics, right: QES, solid black line: the maximum mass allowed by the radius and radial velocity.

If the local geometry of spacetime were flat, then Eq. (23) would give the same result locally as it does globally. After all, the laws of dynamics don’t change with scale. We should observe galaxies where the mass was proportional to the square of the velocity and dependent on the radius, but we do not. Where is the natural evidence for this hypothesis of locally flat spacetime?

Figure 6. The relation between tangential velocity and mass. Blue circles are the combined gas and stellar mass of the gas-rich galaxies and the solid line is the maximum mass allowed for the given velocity according to the dynamics of quadratically expanding space.

A study of the relation between velocity and mass was conducted in (McGaugh 2012) with the assumption that luminosity is an imperfect proxy for mass. Gas rich galaxies are better proxies as they are not as affected by the vagaries of the stellar mass-to-light ratios. Those results are displayed in Figure 6 and overlaid with Eq. (27). The reduced of 0.75 demonstrates that the QES model accurately predicts the Baryonic Tully-Fisher Relationship (BTFR). The evidence for locally curved geometry of spacetime is written plainly in the night sky.

# Cosmic Microwave Background Radiation

The oldest light we can observe is the Cosmic Microwave Background (CMB) radiation. Quantum fluctuations in the primordial plasma set up density waves that travelled at a known velocity. When the temperature of the early universe fell far enough through adiabatic cooling, the photons decoupled from the plasma and streamed freely across the universe. When the photons were no longer scattered by electrons, the density waves collapsed, and the photons carried away with them a record of how far the waves travelled: the sound horizon. This picture of the early universe contains information about the geometry of spacetime.

The primary observable from this surface of last scattering is the angular scale which is a measurement of the characteristic wavelength of the harmonic series found in the power spectrum of this light. The angular scale informs us about the geometry by relating the curvature of spacetime and the distance the light has travelled to the sound horizon. Using the metric from Eq. (13) to solve for the angular scale, , and observing that the time, d, distance, , and the azimuth, , do not change when measuring this angle, we have:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  | (28) |

Where is the comoving sound horizon, is the curvature of spacetime and is the comoving distance from the observer to the surface of last scattering. The first Friedman equation provides us with a method to calculate the curvature from the particle density:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Where is the present-day density of baryons, photons, and neutrinos.

|  |  |  |
| --- | --- | --- |
|  |  |  |

Solving for the present day when and , we have:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | () |

The sound horizon can be calculated from the photon and baryon densities as:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | () |

Where is the baryon-photon momentum density ratio, and is the conformal time of last scattering. The particle horizon, , is the maximum distance that a massless particle can travel to an observer since the start of time. The conformal time is the time that this trip would take. We can calculate the conformal time from the limit of Eq. (15) as:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | () |
|  |  |  |
|  |  | () |

The present-day photon density is well known from the temperature of the CMB:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  |  |

Where is the current CMB temperature which we have taken from (Fixsen 2009) as 2.7255K, is the radiation density constant, is the Boltzmann constant, and , is the Planck constant. In addition to the photon density, the photon number density is calculated here as a reference. The baryon density is unknown as is the conformal time of last scattering, , which is defined as the conformal time when the optical depth indicates just one Thomson scattering from emission to observation (Ade et al. 2014). The optical depth, , is given by:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | (35) |

Where is the electron number density, is the free electron fraction, is the abundance of hydrogen, and Y is the abundance of helium, and are the masses of hydrogen and helium, respectively. For the optical depth formula, , in Eq. (35), is the present-day conformal time and is the Thomson cross section.

Figure 7. The free electron fraction history. blue: ΛCDM with (Planck Collaboration et al. 2019) parameters, red: QES with and .

The ionization fraction, , is calculated from the RecFast++ code, originally developed by (Seager et al. 1999) and enhanced by the work of (Chluba & Thomas 2011), (Rubiño-Martín et al. 2010), and (Chluba et al. 2010), and produces a history of the free electron fraction based on a model of expansion (see Figure 7). The code was modified to calculate the Hubble function as:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  |  |

The code was also modified to accept the baryon density directly rather than as a fraction of the critical mass. By iteratively solving equations (28) and (35) for from (Planck Collaboration et al. 2019) and feeding the solution for back into RecFast++, we find that . The conformal time of last scattering, is , which puts the cosmological time at 781 Myr and the redshift at . The sound horizon, , is , the distance to the surface of last scattering, , is . Because it was a condition of the simultaneous equations, the angular scale, , is exactly . The radius of curvature is:

|  |  |  |
| --- | --- | --- |
|  |  | () |

Note that we have balanced the books on the composition of the universe with constituents that are easily produced in a laboratory. On average, the universe contains roughly four ordinary atoms and half a billion photons in a cubic meter and some still uncounted number of neutrinos, but nothing terribly exotic.

# Baryonic Acoustic Oscillation

The collapse of the primordial density wave not only left an imprint on the background radiation, but it also left overdense regions of matter that is observed today as a faint bump in the correlation function of the distances between galaxies.

A study was performed by (Eisenstein et al. 2005) on a sample of 46,748 galaxies from the Sloan Digital Sky Survey and found a peak in the bump on the correlation function at . This translates to a present-day sound horizon of using the parameters from (Planck Collaboration et al. 2019) (). The assumptions of ΛCDM are integral to this study and so the conclusions are model-specific. The proper approach to using this data would be to rework the entire SDSS data set using the QES model, but that effort is beyond the scope of this document. However, our model predicts a comoving sound horizon of , so, our prediction, if not yet confirmed by a study of BAO, is not excluded by a superficial comparison.

# HOrizons

The path that a photon takes informs us about the topography of the universe through which it travels. If there is no path between the emitter and the observer, we say this particle lies beyond our horizon and that horizon becomes a useful landmark on our map. Figure 8 describes these horizons using a spacetime diagram and allows us to better visualize the expansion of space.

If two particles lie beyond the particle horizon, then there is no opportunity for them to exchange information about the conditions in their part of space, such as the average temperature of a photon. We would expect two parts of space that have never had a chance to exchange information to have different conditions: for example, different temperatures. If we examine the spacetime diagrams in Figure 9, we can make some qualitative predictions about the background radiation expected from the different models.

Figure 8. The spacetime diagrams for left: ΛCDM, and right: QES. Green: Hubble Sphere, blue: Particle Horizon, orange: light cone and magenta: present-day world line, black: here and now of the observer. The dotted lines represent constant comoving distances (world lines) at regular intervals.

The geometry of ΛCDM does not allow enough time for the background radiation to thermalize. The particle horizon at the time of last scattering was which corresponds to a comoving distance of . In this model, the distance to the surface of last scattering is , so we would expect regions separated by in the sky to be out of causal contact with each other, and therefore have different temperatures. In Figure 9 (left) we see that the light cone corresponding to two photons from opposite points on surface of last scattering have never been in causal contact. ΛCDM, then, predicts an inconsistent, lumpy universe, but that is not what we observe.

Figure 9. The spacetime diagrams emphasising the early universe for left: ΛCDM, and right: QES. black dashed line: the time when the CMB photons were last scattered, blue: the particle horizons corresponding to two points separated by the comoving distance from the observer to the surface of last scattering, orange: the light cone of CMB photons, black: the here and now of the observer.

In the geometry of quadratically expanding space, the particle horizon at the time of last scattering is , or in comoving coordinates. The comoving distance to the surface of last scattering is , so photons have had more than enough time to exchange information. The QES model predicts complete thermalization of the background radiation with a horizon that is more than 5 times the circumference of the universe. All objects in the QES universe are causally connected. Examination of the CMB radiation from (Wright 2003) tells us that the temperature of the observable universe varies only one part in 150,000, confirming this prediction.

Advocates of ΛCDM will argue that exotic physics gives the universe enough time to thermalize before the last scattering. However, when a model predicts an infinite number of universes, one can safely argue that the hypothesis is no longer disprovable. Here we have demonstrated a simple, geometric principle that predicts a single universe that is homogenous and isotropic.

# Conclusion

There are three imaginary dimensions of time and three real dimensions of space. Space is the cross product of time and, thus, expands quadratically with time. If an observer experiences all three dimensions of time passing at the same rate, then objects will appear to move irresistibly away from each other at a constant acceleration of . This model of expansion is strongly confirmed in the relation between redshift and brightness in type Ia supernovae.

For decades scientists have been searching for the reason behind the unexpectedly high velocity of stars in spiral galaxies and galaxies in clusters. Unable to fix the old physics, they began inventing new and increasingly exotic physics. We have shown here that ordinary matter is sufficient to describe the velocity curves in spiral galaxies if we set aside our naïve and untested assumption that the geometry of spacetime is locally Lorentzian.

A geometry of constant and quadratic expansion informs us that the universe is a 3-sphere and has a circumference of . This geometry is in general agreement with the distribution of galaxies on large scales and predicts the isotropic universe we observe. With this geometry we have a complete inventory of the universe using just the particles predicted by the Standard Model of Particle Physics.

Entities should not be multiplied beyond necessity. Here, we have replaced several arguably unscientific hypotheses with a single geometric principle that is a much better match to the data on every scale. If the reader is not persuaded with parsimony and still wants proof, then look at the night sky. The Tully-Fisher relation is irrefutable evidence that objects at rest accelerate.

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